Counting Statistics and Concentration Error Bars

Spectradyne reports its calculated concentrations as a function of particle size from the raw, individual particle counting and sizing process. The concentrations calculated from these data therefore have an intrinsic uncertainty due to counting statistics. Here we briefly discuss how we generate the error bars in our concentration plots, based on these statistics.

The number of events that occur in a given particle size interval, if uncorrelated and from a parent distribution having constant mean value μ, is controlled by the Poisson distribution function $P(\mu,k)$, which gives the probability P that k events will occur in the measurement interval. The distribution is given by

$$P(\mu,k)=\frac{\mu^k e^{-\mu}}{k!}$$

This has its maximum value at $k=\mu$, and for large μ approaches the normal (Gaussian) distribution.

Here the parent distribution's mean value μ is unknown, and the counting experiments are used to determine its value. A single counting measurement that yields a value N of course does not determine the mean μ, as many values of μ could yield the same N counts in any given experiment. However, the most likely value is $\mu=N$, as that corresponds to the parent distribution that is most likely to return N counts. The uncertainty $\Delta\mu$ in assigning this value to μ is determined by finding the one-sigma range, from $\mu-\Delta\mu$ to $\mu+\Delta\mu$, for which the probability of capturing the correct value is $\mathrm{erf}\,(1/\sqrt{2})$ or 0.67289 … (where erf represents the error function). In other words, $\Delta\mu$ corresponds to a confidence level of capturing the correct value for the mean of one sigma.

The probability $P(\mu,N)d\mu$ is the probability that the N counts came from a parent distribution with mean between μ and $\mu+d\mu$. The uncertainty $\Delta\mu$ is then calculated by satisfying the integral equation

$$\int_{N-\Delta\mu}^{N+\Delta\mu} P(\mu,N)d\mu=\mathrm{erf}\left(\frac{1}{\sqrt{2}}\right)=0.672689\ldots$$

For N large, meaning larger than about 10, $\Delta\mu=\sqrt{N}$ turns out to solve this equation quite accurately (to better than 1% for N bigger than 12, to better than 5% for N bigger than 3). For small N the square root rule does not work very accurately, so we can instead use numerical integration to give the following results (we also list the conventional $\sqrt{N}$ for comparison):

N	$+\Delta\mu$	$-\Delta\mu$	$\sqrt{N}$
0	1.148	0	0
1	1.36	-1.00	1
2	1.57	1.57	1.41
3	1.84	1.84	1.73

Note that the error bars are not symmetric for small N, and we also assign an error to a zero-count result. This table, combined with the simple expression for larger N, provides the one-sigma estimate for the uncertainty in setting $\mu=N$.

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